

# INCREASING GEOMETRIC LITERACY SKILLS OF FUTURE MATHEMATICS TEACHERS THROUGH 3D PRINTING ACTIVITIES IN GEOGEBRA ENVIRONMENT

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## Abstract

Integrating digital technologies into education, particularly in the area of geometric literacy, is transformative. GeoGebra's dynamic mathematics software facilitates interactive learning of geometry and algebra that is consistent with constructivism's principles. Utilizing 3D printing further enriches the experience; it allows for a hands-on approach to understanding geometric concepts combined with manipulative activities and activities aimed at understanding mathematical concepts. Geometry became more tangible and engaging when students used GeoGebra to design three-dimensional representations of geometric figures. These designs were then brought to life through 3D printing, allowing students to interact with their geometric creations physically. At the same time, it allowed for removing formalism from mathematics education. Since visualization is a key component in developing the ability to interpret and understand geometric shapes, manipulating these 3D models enhanced the learning experience. In our research, we presented a series of activities that both enhance spatial understanding and geometric reasoning and promote creativity and personal expression. Such activities were particularly beneficial for our students - future mathematics teachers. In addition to providing valuable technology skills, including 3D printing activities in the geometry curriculum helps prepare teachers to use this innovative tool in their future teaching, thus improving their students' geometry education. Our observations support the suitability of 3D printing as a tool to promote geometric literacy and demonstrate its potential to improve spatial perception and understanding in geometry.

**Keywords:** 3D printing, *GeoGebra*, geometric literacy.

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## 1. Introduction

The integration of digital technologies into education is a transformative process, reshaping traditional pedagogical approaches and enhancing the learning experience across various domains. In the context of geometric literacy, the use of digital tools such as GeoGebra and 3D printing represents a significant advancement. GeoGebra, a dynamic mathematics software, supports interactive and visual learning of geometry and algebra. Its alignment with constructivist principles underscores its ability to facilitate a deeper understanding of mathematical concepts through active engagement and exploration.

Constructivism, as a learning theory, posits that knowledge is constructed through active participation and reflection on experiences (Piaget, 1972). This theoretical framework emphasizes the importance of learners being actively involved in the learning process, rather than passively receiving information. GeoGebra's interactive features enable students to manipulate geometric figures, observe the effects of transformations, and explore algebraic relationships dynamically. This hands-on approach aligns with constructivist ideals, promoting a deeper and more meaningful understanding of geometry (Hohenwarter & Lavicza, 2007).

The incorporation of 3D printing technology further enhances this learning paradigm by providing a tangible, hands-on dimension to geometric exploration. 3D printing allows students to design and create physical models of geometric shapes, bridging the gap between abstract concepts and concrete understanding. This tactile experience is particularly valuable in mathematics education, where visualization and spatial reasoning play crucial roles in comprehending geometric relationships (Cohen et al., 2015).

Visualization is a critical component in the development of geometric literacy. It involves the ability to interpret and understand the properties and relationships of shapes and spaces (Bishop, 1980).

By enabling students to interact with three-dimensional models, 3D printing enhances their spatial perception and geometric reasoning. The physical manipulation of these models facilitates a deeper engagement with geometric concepts, making the learning process more interactive and engaging (Canessa et al., 2013).

In the context of training future mathematics teachers, incorporating 3D printing into the geometry curriculum offers dual benefits. It not only equips them with valuable technological skills but also prepares them to integrate innovative tools into their teaching practices. This forward-thinking approach ensures that future educators are well-prepared to enhance their students' geometry education using advanced technological tools (Chrysanthou, 2018).

Our research highlights a series of activities designed to enhance spatial understanding, geometric reasoning, creativity, and personal expression. These activities have proven particularly beneficial for our students, who are prospective mathematics teachers. By engaging with 3D printing, they develop a hands-on approach to teaching geometry, which in turn has the potential to improve their students' learning outcomes (Martínez et al., 2020).

## 2. Design, Objectives and Methods

### 2.1. Design and Objectives

This pilot study aimed to integrate theoretical knowledge with practical application in the calculation of the volume of 3D objects using planar sections. The activities were inspired by Dr. Diego Lieban from Instituto Federal de Educação, Ciência e Tecnologia do Rio Grande do Sul, Brazil, and aimed to bridge the gap between theoretical mathematics and practical application through the use of 3D printing technology. During the winter term of the 2023/2024 academic year at the University of Ostrava, the research involved seventeen second-year bachelor's degree students specializing in teaching for the second grade of elementary schools. The objectives of these activities were:

- ✓ Connect theoretical knowledge of volume calculation with practical problems involving planar sections.
- ✓ Foster creativity, spatial visualization, and logical thinking.
- ✓ Enhance digital competencies using 3D printing technology.
- ✓ Create educational aids to be used in teaching practice, demonstrating the practical application of mathematical concepts.

### 2.2. Methods

The activities were structured into three main teaching sessions:

**2.2.1. Theoretical Introduction.** The first session focused on explaining the principles of volume calculation using planar sections. The teacher provided examples of complex printed objects and guided students through solving real-life volume calculation problems. Emphasizing collaborative learning, students worked together to discover solutions. The goal was to motivate students to creatively apply their theoretical knowledge and to introduce the use of 3D printing in mathematics education.

**2.2.2. 3D Modeling.** In the second session, students worked in pairs to design and model their 3D objects using GeoGebra 3D software. Each pair created a 3D object composed of several basic geometric shapes. This session aimed to enhance students' skills in spatial visualization and geometric modeling. For those lacking experience with GeoGebra 3D, the teacher demonstrated the modeling process using a simple example before allowing students to work independently. The teacher provided individual consultations as needed to support the students' learning.

**2.2.3. Practical Application and 3D Printing.** The final session involved using PrusaSlicer software to prepare the models for 3D printing. Students exchanged their designs as .ggb files, converted them into .stl files, and imported them into PrusaSlicer. They adjusted the models by scaling, copying, and rotating them to facilitate planar sections suitable for volume calculation. The teacher assisted with PrusaSlicer usage, ensuring students correctly prepared their models for printing. After slicing, the .gcode files were printed on a 3D printer. During printing, students discussed the accuracy of their sectioning methods and volume calculations. This session provided practical 3D printing experience and reinforced theoretical knowledge application.



Figure 1. Students observing the printing of their 3D models/table.

### 2.3. Observations and Data Collection

Throughout the sessions, observational data were collected to assess student engagement, collaboration, and practical application of theoretical knowledge. Observations indicated high levels of engagement and enthusiasm for using 3D printing technology. Students actively collaborated and assisted each other during the modeling and printing processes, demonstrating a strong grasp of the geometric concepts involved.

The study was evaluated using a questionnaire filled out by the students. The questionnaire assessed their understanding of geometric concepts, ability to apply theoretical knowledge practically, and overall experience with 3D printing technology in mathematics education. Observational data provided additional insights into student behavior and learning outcomes.

## 3. Results

Positive feedback from the questionnaires indicated that students successfully combined theoretical knowledge with practical skills. These activities enhanced their understanding of geometric relationships, improved digital competencies, and problem-solving abilities. The 3D printed objects served as valuable teaching aids, demonstrating the practical application of mathematical concepts. Observational data supported these findings, showing high levels of student engagement and collaboration.

## 4. Discussion and Conclusions

The pilot study demonstrated the benefits of integrating 3D printing technology into mathematics education. It provided students with a deeper understanding of geometric concepts, enhanced their digital literacy, and offered practical experience with advanced technology for educational purposes. The positive outcomes suggest that similar approaches can be beneficial in other areas of mathematics and science education.

In conclusion, the integration of GeoGebra and 3D printing into geometry education represents a significant step forward in promoting geometric literacy. These tools not only enhance spatial perception and understanding but also foster creativity and innovation in mathematical thinking. Our observations support the suitability of 3D printing as a relevant tool for improving geometric education, highlighting its potential to transform the teaching, and learning of geometry (Cohen et al., 2015).

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