Co-funded by the European Union

3D STEAM

Project title: ***Accelerating STEAM-related Knowledge and Skills via 3D Modelling and 3D Printing*** *(reg. no. 2023-1-CZ01-KA220-HED-000160664)*

Programme: Erasmus+ KA220-HE - Cooperation partnerships in higher education

MODEL ACTIVITY 1

Section of a cube

Available in: **English**

(2025)

Leading organization: University of Jyväskylä (FIN)

Partner organization: Johannes Kepler University Linz (AT)

University of Ostrava (CZ)

University of Luxembourg (LUX)

Comenius University Bratislava (SK)

***Co-funded by the European Union**. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the National Agency. Neither the European Union nor the grant provider, the Czech National Agency for International Education and Research, can be held responsible for them.*

OSTRAVSKÁ UNIVERZITA

JOHANNES KEPLER UNIVERSITÄT LINZ

UNIVERSITY OF JYVÄSKYLÄ
FINNISH INSTITUTE FOR EDUCATIONAL RESEARCH

UNIVERSITÉ DU LUXEMBOURG

COMENIUS UNIVERSITY BRATISLAVA

Teaching Planning Tool – 3D Modeling and 3D Printing (STEAM)

3D Printing and Modeling as a Tool for the Creative Development of Students' Knowledge and Skills (Original Designs, Iterations)

PART 1: Conceptual Planning (General Information)

Title of the (STEAM) Thematic Unit

Intersection of a Plane with a Solid – Section of a Cube

Authors (Names / School / Country): Martina Totkovičová, GJP, Vazovova 6, Bratislava (Faculty of Education, Comenius University, Bratislava), Slovak Republic

SUBJECT	Mathematics, Informatics
GRADE	2. High school grade (age 16–17 years)
NUMBER OF LESSONS	6 lessons of 45 minutes each (4 mathematics + 2 informatics) total 270 minutes
DIDACTIC OBJECTIVES	• To develop spatial imagination and the ability to visualize geometric situations in various contexts (manipulative, digital, theoretical), • To develop spatial thinking and understanding of the concept of the intersection of a plane with a solid through practical 3D modeling, • To develop digital competencies through working with 3D modeling software, • To use formulas to calculate the surface area of various plane figures formed by the intersection of a cube with a cutting plane, • To investigate how known points belonging to the section can influence its shape, and how many polygons may thus be formed, • To develop creative and logical thinking through solving non-standard geometric problems, • To foster collaboration and communication within the group in argumentation and justification of solutions, • To support the transfer of knowledge between the theoretical and practical dimensions of geometry, • To develop the capacity for autonomous learning and constructive engagement with error.

SPECIFIC OBJECTIVES	• To become familiar with GeoGebra 3D and TinkerCAD tools. • To create and modify 3D solids. • To compare theoretical calculations with real-world models. • To gain confidence in the use of 3D modeling and printing tools. • To strengthen logical and spatial thinking. • To realize the interconnection of mathematics, design, and technology.

THEMATIZATION AND PROBLEMATIZATION

The topic is meaningful in terms of connecting mathematical reasoning with students' experiences in the context of real-world models and digital technologies. The solved problem — the section of a cube by a plane defined by three points — constitutes a non-trivial stereometric task in which students develop spatial imagination, logical reasoning, and the ability to connect various representations (model, sketch, digital 3D object).

Key questions:

- How can the intersection of a plane with a cube be determined?
- What shapes of sections can arise?
- How can the correctness of the solution be verified using both a physical and a digital model?
- What are the differences between working with a physical model and a digital environment?

The didactic strategy is grounded in constructivism and the principles of learning through problem-based learning (PBL). Students become active participants in the learning process—they formulate the problem, seek solutions, test hypotheses, and reflect on their own understanding. The teacher's role is to guide their thinking through questions and to create conditions for inquiry, experimentation, and collaboration. Within PBL, emphasis is placed on the process of discovery rather than merely on the outcome.

The EUR method (Elicitation – Realization of Meaning – Reflection) was also employed:

- Elicitation: students predict what shape may be formed by the intersection of a plane and a cube,
- Realization of meaning: students investigate the situation experimentally (models made from skewers and straws, or 3D visualization in GeoGebra and TinkerCAD),
- Reflection: students compare different approaches, articulate the connections they have discovered, and assess which approach helped them understand the solution most effectively.

An activity conducted in this manner leads students to a deeper understanding of geometric concepts and principles, integrates knowledge from mathematics, physics, and technology, and illustrates the purpose of learning in the context of a real-world problem.

STUDENT ENTRY SKILLS

☐ Fundamentals of TinkerCAD

☐ Experience with GeoGebra 3D

☐ Working with STL/OBJ files

☐ Fundamentals of slicing (PrusaSlicer/Cura/Bambu Studio)

☐ Experience with other software (please specify) ....

☐ Safety in 3D printing

3D TOOLS AND SOFTWARE

• Modeling software: TinkerCAD, GeoGebra 3D

• Slicer: PrusaSlicer

• Viewer/AR:

3D PRINTING

Equipment required for the activity: paper straws, hot glue gun/superglue, cardboard, cardstock, 3D printer

Educational objectives related to 3D modeling and 3D printing

Main objective:

To develop students' spatial and constructive reasoning through the creation, analysis, and interpretation of 3D models that integrate theoretical knowledge from solid geometry with the practical application of 3D technologies.

Secondary objectives:

- Enumerate the fundamental phases of the process for creating a model intended for 3D printing.
- Identify the basic geometric solids and their properties essential for model construction.
- Explain how the geometric shape changes in relation to the position of the section plane.
- Describe the relationship between a digital model, a real object, and its mathematical representation.

- Use software tools (GeoGebra 3D, TinkerCAD) to create and modify geometric models.
- Apply knowledge of planes and intersection in a practical task—designing a section of a cube defined by three points.
- Compare different approaches to solving the task (manipulative model, digital model, computational solution).
- Identify errors or inaccuracies in the model design and propose methods for their correction.
- Evaluate the quality and accuracy of your own model in comparison with the theoretical solution.
- Discuss the benefits and limitations of 3D technologies in mathematics education.
- Design and create an original 3D model of a solid section according to specified parameters.
- Experiment with various shapes and positions of the plane to discover new relationships and connections.
- Develop a procedure for classmates or younger students—'How to create a section of a cube with a plane'—as a form of knowledge sharing.

STEAM elements

STEAM ELEMENT	CHARACTERISTICS OF THE ELEMENT IN THE ACTIVITY
SCIENCE	
TECHNOLOGY	Students use digital tools—**GeoGebra 3D**, **TinkerCAD**, and **PrusaSlicer**—to create, modify, and prepare models for 3D printing. They learn to export models in G-code format and to understand the connection between the virtual and physical environments.
ENGINEERING	During the design and testing process, students apply principles of engineering thinking: planning, verifying accuracy, correcting errors, and optimizing model shapes. They learn to understand a 3D model as a functional output, not merely as a visualization.
ART	This activity fosters aesthetic perception and creativity—students experiment with various materials while creating models, design visually appealing solutions, and discuss the representativeness of individual models. They perceive the precision of execution and proportions not only mathematically but also visually.
MATHEMATICS	Students apply knowledge from geometry and solid geometry—they determine a plane in space, analyze the intersection of planes and cubes,

	and use concepts such as vertex, edge, face, parallelism, and intersection. They integrate various forms of representation (sketch, physical model, digital model) and develop spatial reasoning.

Syllabus of Integrated Subjects within the Curriculum

SUBJECTS	EDUCATIONAL AREA	CONTENT STANDARD	PERFORMANCE STANDARD
Informatics / Digital Technologies	Man and Technology	The student uses digital tools for modeling, simulation, and 3D visualization of objects.	The student can create a simple 3D model in the TinkerCAD environment, export it in STL format, and prepare it for printing using a slicer.
Technology / Physics	Man and the World of Work	The student understands the fundamental principles of construction, stability, and accuracy of technical models.	Is able to plan and implement a simple engineering project—from initial design to verifying the functionality of the created model.
Art Education / Art and Culture	Art and Culture	The student explores the shapes and proportions of objects, applying aesthetic principles such as symmetry, balance, and composition.	Is able to design a visually appealing 3D model that integrates functionality with aesthetic expression.
Mathematics	Mathematics and Information Processing	The student understands the concept of planes in space and the relationship between a plane and a solid, and analyzes the intersection of a plane and a cube.	Can determine the intersection of a plane and a cube defined by three points, and create its sketch and model in a digital environment.

PART 2: Specific Planning – Teaching Unit

Title of the Teaching Unit (1.): Introduction to the Topic of Sections of the Cube

Lesson Objectives: To become familiar with the basic properties and rules. Connecting 3D models with sketches in free parallel projection

Learning outcomes for students: Creating a cube model using a Polydron kit and actively utilizing it to discover the properties of skew and intersecting lines in free parallel projection.

Lesson Plan (Phased Description)

- Introductory Motivation – short video https://www.youtube.com/watch?v=WQgw89fT4BM as well as a visual of the intersection of a cube with simple planes using a model.
- Discussion: In which practical contexts do we encounter intersections of planes (architecture, 3D modeling, design).
- Work with Polydron – students assemble a cube and investigate relationships among edges (skew, intersecting) and lines lying within the faces of the cube.
- Projection onto a plane – attempt to sketch the cube in free parallel projection, including marked lines.
- Discussion on why some lines 'appear' to intersect, even though they do not intersect in space.

Assessment – overview

Observation of students' work with the model, ability to identify the properties of lines, participation in the discussion.

Reflection/assessment – feedback:
Students identified the problem of spatial representation and recognized the limitations of a 2D sketch. Most students understood that oblique parallel projection 'distorts' spatial relationships.

Title of the teaching unit (2): Section of a cube by a plane defined by three points

Lesson objectives: To construct an original model that can be used to extend the individual edges of cubes, in order to determine whether given lines in space intersect—whether they are skew lines, or whether they do not intersect and their apparent intersection is merely an 'illusion' caused by free parallel projection in the plane.

Student outcomes: Construction of a cube model using paper straws and its active application in exploring the intersection of a plane defined by three points located on the edges of the cube. Determining the shape of this intersection, followed by a discussion on what type of polygon the intersection of a plane with a cube may form.

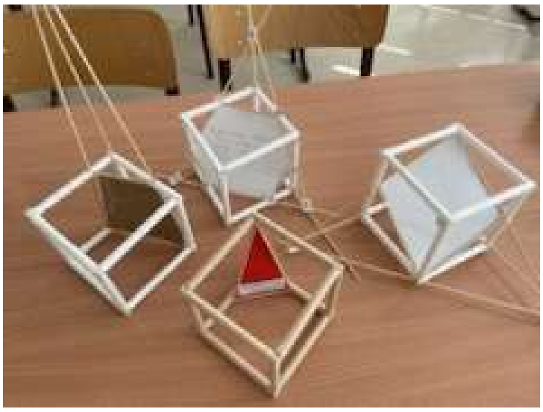
Lesson Plan (Phased Description)

- A brief review: what is a plane, and how can it be defined.
- The teacher assigns three points, each located on a different edge of the cube.
- Students construct a cube using straws and skewers.
- Experimental determination of the intersection of a plane with a cube—marking the intersections on the models.
- Discussion: the shape of the intersection and the number of vertices (which polygon is formed).
- Group presentations of different solutions.

Assessment – overview

Assessment – Overview:
Activity in groups of three, ability to argue and identify the resulting shape, and model accuracy.

Reflection/assessment – feedback:
Students were motivated to engage in practical work. They discovered that the same task can lead to different visualizations, which stimulated a discussion on the precision of geometric constructions.



Teaching Unit Title (3.): Determining the Content of a Cube Section

Lesson Objectives: To determine the exact dimensions and surface area of the section.

Student Outcomes: Creation of a cube section model from cardboard. Modeling the given situation in GeoGebra 3D and verifying calculations using GeoGebra 3D.

Lesson Plan (Phased Description)

- Revision: formulas for surface area and the Pythagorean theorem; application of similarity in computational geometry.
- Creation of a cardboard model of the section based on the straw construction method.
- Measurement and calculation of the area of the intersection.
- Working in GeoGebra 3D – modeling the cube and planes, and verifying the shape and dimensions of the section.
- Comparison of calculations with the digital model.

- Discussion of the advantages of digital verification of results.

Assessment – overview

Assessment – Overview:
Accuracy of calculations, ability to transfer the model into GeoGebra, and argumentation when making comparisons.

Reflection/assessment – feedback:
Students appreciated the visualization in GeoGebra. They discovered that not all models matched the calculations, which led to a better understanding of the need for precision.

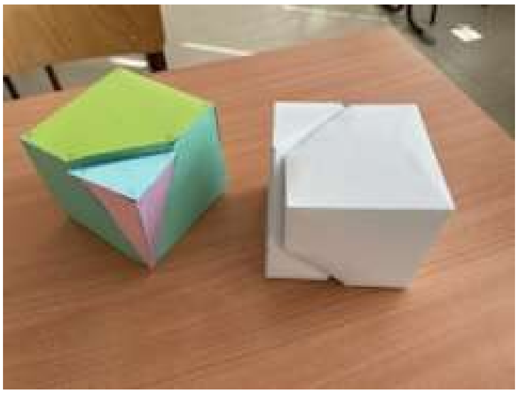
Lesson Title (4.): **Nets of solids resulting from cutting cubes with a plane**

Lesson objectives: To create nets of solids that result from cutting a cube with a plane.

Student outcomes: Creation of two paper models to serve as prototypes for work with 3D printing.

Lesson Plan (Phased Description)

- Discussion: What happens when a cube is 'cut'?
- Designing nets of the resulting solids on paper—students consider which edges are shared.
- Creating paper models of both solids.
- Comparing how they can be reassembled into the shape of the original cube.
- Reflection: Which shapes were easier to model and which were more difficult.



Assessment – overview

Assessment – Overview:
Construction of nets, accuracy of calculations and assembly, and the ability to justify the chosen approach.

Reflection/assessment – feedback:
Students realized that each net represents a different interpretation of the same problem. Spatial imagination and patience when working with materials were developed.

Title of the lesson unit (5, 6.): Introduction to and work with the TinkerCAD program

Lesson objectives: To familiarize students with the functionalities offered by the TinkerCAD program to such an extent that they are able to design solids resulting from the intersection of cubes with a plane.

Student outcomes: Creation of two 3D models of solids using 3D printing, which will serve as output products.

Lesson Plan (Phased Description)

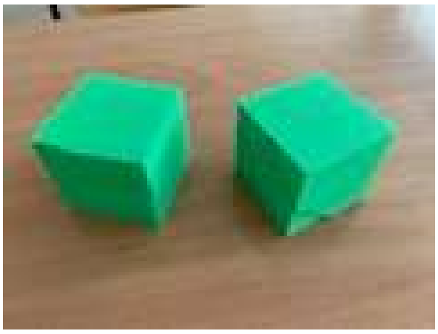
- Introduction: Explanation of the objective – to transfer a model from paper to digital form.
- Overview of TinkerCAD functions and demonstration of basic tools.
- Brainstorming: how to connect two solids to create a cube.
- Proposed solution with a 'cylindrical lock'—recess and protrusion.
- Independent student work on the design, STL export, slicing, and 3D printing.
- Final comparison of physical models and discussion on the accuracy of printing.

We devoted considerable time to brainstorming how to model the entire structure so that both solids could be joined to form a cube. We encountered this problem when creating the paper model and attempted to solve it. The concept we ultimately realized was to create a cylindrical recess in the plane of intersection in one solid, and at the same location in the other solid, to model a cylinder from the plane of intersection that 'fits' into the recess, thereby preventing slippage.

Assessment – overview

Assessment – Overview:
Successful completion of the model, correct export to STL, understanding the principle of slicing, and functionality of the printed model.

Reflection/assessment – feedback:
Students perceived the integration of mathematics and technology as meaningful. They discovered the importance of precise planning and accurate dimensions in 3D modeling. Several individuals expressed surprise at the extent to which mathematical knowledge must be applied in technical practice.



Assessment of 3D Modeling and 3D Printing (Detail)

1) Does the student understand / is the student able to…?
The student understands the relationships between a real object and its digital model. The student can explain how mathematical principles (planes, intersections, dimensions) are transferred into the 3D environment. Within the mind map, the student was able to identify the procedures from construction to 3D printing and justify the importance of each step.

2) Did the student create …?
– *method: product assessment*
The student created a digital model of the solid using TinkerCAD and a physical model through 3D printing. The product meets both the basic technical and mathematical requirements—dimensions, planarity of the section, and functionality of the lock between solids. In evaluating the product, accuracy, aesthetics, stability, and originality of the solution were assessed.

3) Can the student present …?
– *method: presentation, peer review*
The student was able to present their process from design to printing, identify mistakes, and explain their solutions. Mutual evaluation between groups (peer review) fostered argumentation and critical thinking. Students were able to articulate how their approach differed and reflected on how the process could have been carried out more effectively.

4) Implements / improves …?
– *method: prototype revision*
Based on feedback, the student was able to modify the design, correct errors in dimensions or shapes, and optimize the model for 3D printing. In this way, they adopted the fundamental principle of the iterative process—viewing mistakes not as failures but as a natural part of learning. Most groups were able to redesign their prototype to ensure its functionality.

5) Team collaboration …?
– *method: observation, self-assessment*
Students worked in small teams of three with clearly defined roles (construction, calculation, modeling, printing). Observation revealed that most teams communicated effectively and were able to utilize the individual strengths of their members. In their self-assessment, students reflected on how collaboration influenced the quality of the final model and their own motivation.

Co-funded by the European Union

3D STEAM

Project: Accelerating STEAM-related Knowledge and Skills via 3D Modeling and 3D Printing
Reg. no. 2023-1-CZ01-KA220-HED-000160664

Supplementary information (optional)

Notes: Active collaboration with the computer science teacher is necessary; this teacher should be proficient in operating the 3D printer and experienced in working with TinkerCAD and PrusaSlicer.

Educational resources / supporting materials:

- Mathematics textbook for high schools – thematic unit: *Stereometry, intersections of solids*.
- GeoGebra 3D – software for the visualization of spatial relationships and verification of calculations.
- TinkerCAD – an online tool for 3D modeling and preparing objects for printing.
- Polydron, paper straws, skewers, office supplies (glue, scissors, cardboard) – manipulative aids for constructing physical models.
- PrusaSlicer / Ultimaker Cura – software for converting 3D models to G-code format.
- 3D printer (e.g., Prusa Mini+, Ender 3) – output device for producing physical models.

Evaluation resources and materials:

- Teacher observation sheets – monitoring work processes, collaboration, and problem-solving.
- Student self-assessment sheets – reflection on individual learning, collaboration, and errors.
- Assessment rubric for 3D modeling – accuracy, functionality, aesthetics, and innovation of the solution.
- Peer review form – reciprocal evaluation of presentations and proposed solutions.
- Photographic documentation of the project process and outcomes.

Other sources / materials:

- https://www.researchgate.net/profile/Lilla-Korenova/publication/317850893_Digitalne_technologie_v_skolskej_matematike/links/594edbb3a6fdccebfa5ebe7b/Digitalne-technologie-v-skolskej-matematike.pdf
- Online tutorials for **GeoGebra 3D** and **TinkerCAD** (YouTube, official websites).
- Scholarly articles on the use of 3D printing in mathematics instruction.
- School educational program